Quiz 5

February 8, 2017

Show all work and circle your final answer.

1. Evaluate $\int_0^1 \frac{y}{e^{2y}}\,dy$

$u = y \quad dv = e^{-2y}dy$

$du = dy \quad v = -\frac{1}{2}e^{-2y}$

$= \left[-\frac{1}{2}ye^{-2y} + \frac{1}{2}\int e^{-2y}dy\right]_0^1$

$= \left[-\frac{1}{2}ye^{-2y} - \frac{1}{4}e^{-2y}\right]_0^1$

$= -\frac{1}{2}e^{-2} - \frac{1}{4}e^{-2} + \frac{1}{4} = \boxed{\frac{1}{4} - \frac{3}{4}e^{-2}}$

2. Evaluate $\int \arctan(1/x)\,dx$

$u = \arctan(x^{-1}) \quad dv = dx$

$du = \frac{1}{1+(x^{-1})^2}\cdot(-x^{-2})dx \quad v = x$

$= x\arctan\frac{1}{x} + \int \frac{x}{x^2(1+\frac{1}{x^2})}\,dx$

$= x\arctan\frac{1}{x} + \int \frac{x}{x^2+1}\,dx \quad u = x^2+1,\ du = 2x\,dx$

$= \boxed{x\arctan\frac{1}{x} + \frac{1}{2}\ln(x^2+1) + C}$

3. Evaluate $\int_0^{\pi/4} \tan^5 x \sec^6 x\,dx$

use $u = \tan x$ since we have $\sec^6 x$

$du = \sec^2 x$

$= \int_0^{\pi/4} \tan^5 x \sec^4 x \sec^2 x\,dx$

$= \int_0^{\pi/4} \tan^5 x (\tan^2 x + 1)^2 \sec^2 x\,dx$

$= \int_0^1 u^5(u^2+1)^2\,du = \int_0^1 u^5(u^4+2u^2+1)\,du = \int_0^1 u^9 + 2u^7 + u^5\,du$

$= \left[\frac{1}{10}u^{10} + \frac{1}{4}u^8 + \frac{1}{6}u^6\right]_0^1$

$= \frac{1}{10} + \frac{1}{4} + \frac{1}{6} = \boxed{\frac{31}{60}}$

4. Evaluate $\int \cos^2(3x)\,dx$

$= \int \frac{1}{2}(1+\cos(6x))\,dx$

$= \frac{1}{2}\left[x + \frac{1}{6}\sin(6x)\right] + C$

$= \boxed{\frac{1}{2}x + \frac{1}{12}\sin(6x) + C}$